MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2015/2016

TMT 1181 – MATHEMATICAL TECHNIQUES II

(All sections / Groups)

10 OCTOBER 2015
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5 pages** only including the cover page with **5 Questions** and an **Appendix.**

2. There are **2 Sections**:
   Section A (3 Questions): Attempt **ALL THREE(3)** questions.
   Section B (2 Questions): Attempt **ONE (1)** question **ONLY**.
   All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write your answers in the Answer Booklet provided, and **start each question on a new page.**

4. Show all steps.

SECTION A

Answer ALL THREE (3) Questions.

Question 1

a. Identify the ***linearity*** and ***order*** of the following differential equations:

i. $2y''+y'-5=0$ **[1 mark]**

ii. $\frac{d^3y}{dx^3}-y\frac{dy}{dx}=0$ **[1 mark]**

b. A ***first order differential equation*** (DE) is given below:

$$y\frac{dy}{dx}=x^2-x$$

i. Show that the DE is ***separable***. **[1 mark]**

ii. Solve the DE using ***separable method***. **[3 marks]**

c. Given the following DE: $x^2\frac{dy}{dx}+x^3y=x^2$

i. Rewrite the DE into the form of $\frac{dy}{dx}+p(x)y=r(x)$. Identify $p(x)$ and $r(x)$.

[2 marks]

ii. Find the ***integrating factor***, $u(x)=e^{\int p(x)dx}$, of the DE. **[2 marks]**

Continued.....

Question 2

a. Given that the ***roots of the characteristic equation*** for the following ***differential equation*** (DE):

$$ay''+by'+cy=0$$

are $\lambda_1=5$ and $\lambda_2=-1$.

i. Find the values of a, b and c **[3 marks]**

ii. Write down the ***general solution***, y, of the DE. **[1 mark]**

b. Given the ***non-homogeneous linear DE***: $y''+2y'=e^x$

i. Convert the above DE to ***homogeneous DE***. Then solve it to get y_h. **[2 marks]**

ii. Find the ***particular solution***, y_p, for the non-homogeneous linear DE using ***undetermined coefficients method***. **[3 marks]**

iii. Form the ***general solution***, $y=y_h+y_p$. **[1 mark]**

Question 3

a. Determine the ***Laplace transform*** for the following functions:

i. $f(t)=e^{2t}+t$ **[1 mark]**

ii. $f(t)=\cos 7t+2$ **[2 marks]**

b. Given that the DE: $y''+4y'-5y=1$.

i. Identify the ***linearity*** and ***order*** of the DE. **[1 mark]**

ii. Show that the ***Laplace transform*** of $y(t)$ is $Y(s)=\dfrac{1}{s(s-1)(s+5)}$ if $y(0)=0$ and $y'(0)=0$. **[3 marks]**

iii. The ***partial fraction*** of $Y(s)=\dfrac{1}{s(s-1)(s+5)}$ is given in the form of $\dfrac{-1/5}{s}+\dfrac{1/6}{s-1}+\dfrac{1/30}{s+5}$. Find the ***inverse Laplace transform***, $L^{-1}[Y(s)]$. **[3 marks]**

Continued.....

SECTION B

Answer ONE(1) Question ONLY.

Question 4

Given the following ***divided difference* (*dd*)** table of a function f.

x_k	y_k	1st *dd*	2nd *dd*	3rd *dd*
0	1			
1	2.7183			
2	10.2175			
3	25.2817			

a. Complete the divided difference table. (**Note**: Round all the numbers to four decimal places.) **[4 marks]**

b. Find the ***cubic Newton polynomial*** $P_3(x)$. **[4 marks]**

c. Approximate $f(1.5)$. **[2 marks]**

Question 5

a. Given the following linear system of equations:

$$4x - y + z = 4$$
$$x + y + 3z = 3$$
$$x - 3y + z = 2$$

i. Write the system above in ***matrix notation***. **[1 mark]**

ii. Represent the system above in the ***augmented matrix***. **[1 mark]**

b. Refer to the linear system of equations in (a).

i. In what condition the system in (a) can be solved using ***Jacobi method***? **[1 mark]**

ii. Does the system need to be rearranged into the form of a diagonally dominant system if it is to be solved by using ***Jacobi method***? Then, solve the system using ***Jacobi method*** by performing the first **TWO iterations**. Given $x_0 = 0, y_0 = 0$ and $z_0 = 0$. (**Note**: Round all the numbers to four decimal places or leave your answers in fraction.) **[7 marks]**

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APPENDIX

LAPLACE TRANSFORMS OF SOME ELEMENTARY FUNCTIONS

Function, $f(t)$	Laplace transform, $F(s)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$e^{\alpha t}$	$\frac{1}{s-\alpha}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$f'(t)$	$s \cdot F(s) - f(0)$
$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$

General solution of the homogeneous linear second order differential equation

For Distinct real roots: $y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

For Repeated roots: $y = (c_1 + c_2 x) e^{\lambda x}$

For Complex conjugate roots: $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Newton Polynomial

$$P_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \ldots + a_n(x - x_0)(x - x_1)..(x - x_{n-1})$$

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